# IMPRS LECTURE DAY: GQFi
20 NOV 2017
EXERCISES

1. CONFORMAL SYMMETRY IN $(1,d-1)$-DIM.

$$= SO(2,d) = \text{POINCARÉ} + \underset{\underset{x^\mu \to \lambda x^\mu}{\parallel}}{\text{DILATATION}} + \underset{\underset{x^\mu \to \frac{x^\mu}{x^2}}{\parallel}}{\text{INVERSIONS}}$$

= COORD. TRAFOS PRESERVING $\eta_{\mu\nu} dx^\mu dx^\nu$ UP TO AN OVERALL FACTOR

DERIVE AdS GEOMETRY AS AN HYPERBOLOID IN $\mathbb{R}^{2,d}$:

$$-\left(\tilde{X}^{-1}\right)^2 - \left(\tilde{X}^{0}\right)^2 + \left(\tilde{X}^{1}\right)^2 + \ldots \left(X^d\right)^2 = -L^2 \leftarrow \begin{array}{l}\text{MANIFEST}\\ SO(2,d)\text{-INV.}\end{array}$$

HINTS:

- (A) USE SINH AND COSH IDENTITY $\downarrow$ PENROSE DIAGRAM
- (B) USE $\tilde{X}^\mu \sim L \frac{x^\mu}{z}$ $\downarrow$ POINARÉ PATCH
- (C) USE $X^0 = \xi \sinh \frac{t}{L}$, $X^1 = \xi \cosh \frac{t}{L}$ $\downarrow$ RINDLER AdS

(D) WHAT IS THE BDRY IN EACH CASE?

(E) WHAT IS THE VOLUME OF CONST. t-SLICE OF AdS ?

ASK TA's WHY THIS BECAME IMPORTANT $\gtrsim 2014$.

(2) (CAN BE DONE AT HOME): SHOW, BY FINDING APPROPRIATE DIFFEOMORPHISMS, THAT

$$ds^2 = \frac{L^2}{z^2}\left(dz^2 + dx_\mu dx^\mu\right)$$

IS INVARIANT UNDER $SO(d,2)$.

HINT: $dx_\mu dx^\mu = -dt^2 + d\vec{x}^2$ IS NATURALLY INV. UNDER POINARÉ.

(3) DERIVE VACUUM 2-PT (AT HOME: 3-PT) FUNCTIONS OF SCALAR PRIMARY OPERATORS IN A CFT USING ONLY THE CONFORMAL INVARIANCE.

HINT 1: FOR A CONF. TRAFO. $x \to x'$, (QUASI) PRIMARY SCALAR OPERATOR $\mathcal{O}$ TRASFORMS A

$$\mathcal{O}_j(x) \to \underbrace{\left|\frac{\partial x'}{\partial x}\right|}_{\text{JACOBIAN}}^{\Delta_j} \mathcal{O}_j(x')$$

AND THE VACUUM STAYS INVARIANT.

HINT 2: INSTEAD OF INVERSION, $x^\mu \to \frac{x^\mu}{x^2}$, CONSIDER SPECIAL CONF. TRAFO.:

$$x^M \to \frac{x^M}{x^2} \equiv x' \to \left(x'\right)^M + a^M \equiv \left(x''\right)^M \to \frac{\left(x''\right)^M}{\left(x''\right)^2}$$